ON THE ROLE OF DENSITY INHOMOGENEITY AND LOCAL ANISOTROPY IN THE FATE OF SPHERICAL COLLAPSE

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Abstract

We obtain an expression for the active gravitational mass of a collapsing fluid distribution, which brings out the role of density inhomogeneity and local anisotropy in the fate of spherical collapse.

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1 Introduction

It is well established that density inhomogeneities play an important role in the collapse of a spherical dust cloud. In particular they may lead to the formation of naked singularities [1], in contrast with the homogeneous case leading to a black hole [2]. However, it is unclear what is the physical reason (if any) behind the link between the final fate of collapse and the presence (or absence) of density inhomogeneities.

It is the purpose of this work to explore the ways in which density inhomogeneities may affect the spherical collapse. To do so, we shall obtain an expression for the active gravitational mass (the Tolman mass) containing explicitly a measure of density inhomogeneity (among other factors). Since our expression is obtained for a general (locally anisotropic) fluid, it also contains a term depending on local anisotropy. We shall see that this term plays a similar role to the density inhomogeneity term, thereby suggesting that local anisotropy might also lead to the formation of naked singularities. As a by-product of this study we shall find some expressions linking the Weyl tensor and the mass function to the energy density inhomogeneity and local anisotropy.

The paper is organized as follows. In the next section the field equations and other useful formulae are introduced. In section 3 we introduce the Tolman mass and discuss its physical meaning. In Section 4 we derive an expression for the Tolman mass and evaluate it at the moment the system departs from hydrostatic equilibrium. A discussion of this expression is presented in the last section.

2 Field Equations and Conventions

We consider a spherically symmetric distribution of collapsing fluid, which for completeness we assume to be anisotropic and bounded by a spherical surface Σ. The line element is given in Schwarzschild-like coordinates by

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + sin^2\theta d\phi^2 \right) \tag{1}$$

where ν and λ are functions of t and r . The coordinates are: $x^0 = t$; $x^1 = r$; $x^2 = \theta$; $x^3 = \phi$.

The metric (1) has to satisfy Einstein field equations

$$G^\mu_\nu = -8\pi T^\mu_\nu \tag{2}$$

which in our case read [3]:

$$-8\pi T^0_0 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) \tag{3}$$

$$-8\pi T^1_1 = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) \tag{4}$$

$$\begin{aligned} -8\pi T^2_2 = -8\pi T^3_3 = & - \frac{e^{-\nu}}{4}\left(2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})\right) \\ & + \frac{e^{-\lambda}}{4}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right) \end{aligned} \tag{5}$$

$$-8\pi T_{01} = -\frac{\dot{\lambda}}{r} \tag{6}$$

where dots and primes stand for partial differentiation with respect to t and r respectively.
In order to give physical significance to the T^μ_ν components we apply the Bondi approach [3], i.e we introduce local Minkowski coordinates (τ, x, y, z), defined by

$$d\tau = e^{\nu/2}dt \qquad dx = e^{\lambda/2}dr \qquad dy = rd\theta \qquad dz = rsin\theta d\phi$$

Then, denoting the Minkowski components of the energy tensor by a bar, we have

$$\bar{T}^0_0 = T^0_0 \qquad \bar{T}^1_1 = T^1_1 \qquad \bar{T}^2_2 = T^2_2 \qquad \bar{T}^3_3 = T^3_3 \qquad \bar{T}_{01} = e^{-(\nu+\lambda)/2}T_{01}$$

Next we suppose that, when viewed by an observer moving relative to these coordinates with velocity ω in the radial direction, the physical content of space consists of an anisotropic fluid of energy density ρ, radial pressure P_r and tangential pressure $P_\perp$. Thus, when viewed by this moving observer, the covariant energy-momentum tensor in Minkowski coordinates is

$$\begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & P_r & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\perp \end{pmatrix}$$

Then a Lorentz transformation readily shows that

$$T_0^0 = \bar{T}_0^0 = \frac{\rho + P_r \omega^2}{1 - \omega^2} \tag{7}$$

$$T_1^1 = \bar{T}_1^1 = -\frac{P_r + \rho \omega^2}{1 - \omega^2} \tag{8}$$

$$T_2^2 = T_3^3 = \bar{T}_2^2 = \bar{T}_3^3 = -P_\perp \tag{9}$$

$$T_{01} = e^{(\nu+\lambda)/2} \bar{T}_{01} = -\frac{(\rho + P_r)\omega e^{(\nu+\lambda)/2}}{1 - \omega^2} \tag{10}$$

Note that the velocity in the (t, r, θ, ϕ) system, dr/dt, is related to ω by

$$\omega = \frac{dr}{dt}\, e^{(\lambda-\nu)/2} \tag{11}$$

Outside of the fluid , the spacetime is Schwarzschild,

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right) \tag{12}$$

In order to match the two metrics smoothly on the boundary surface $r = r_\Sigma(t)$, we require continuity of the first and second fundamental forms across that surface. As result of this matching we obtain the well known result

$$[P_r]_\Sigma = 0 \tag{13}$$

The radial component of the conservation law

$$T^\mu_{\nu;\mu} = 0 \tag{14}$$

gives

$$\left(-8\pi T^1_1\right)' = \frac{16\pi}{r}\left(T^1_1 - T^2_2\right) + 4\pi\nu'\left(T^1_1 - T^0_0\right) + \frac{e^{-\nu}}{r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) \quad (15)$$

which in the static case becomes

$$P'_r = -\frac{\nu'}{2}\left(\rho + P_r\right) + \frac{2\left(P_\perp - P_r\right)}{r} \quad (16)$$

representing the generalization of the Tolman-Oppenheimer-Volkof equation for anisotropic fluids [4].

For the next sections it will be useful to calculate the components of the Weyl tensor. Using Maple V, it is found that all non-vanishing components are proportional to

$$W \equiv \frac{r}{2}C^3_{232} \quad = \quad W_{(s)} + \frac{r^3 e^{-\nu}}{12}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) \quad (17)$$

where

$$W_{(s)} = \frac{r^3 e^{-\lambda}}{6}\left(\frac{e^\lambda}{r^2} - \frac{1}{r^2} + \frac{\nu'\lambda'}{4} - \frac{\nu'^2}{4} - \frac{\nu''}{2} - \frac{\lambda'}{2r} + \frac{\nu'}{2r}\right) \quad (18)$$

corresponds to the contribution in the static case.

Finally, let us introduce the mass function, defined by [5, 6]

$$m(r,t) = \frac{1}{2}rR^3_{232} \quad (19)$$

where the Riemann component for metric (1) is given by

$$R^3_{232} = 1 - e^{-\lambda} \quad (20)$$

Then using the field equation (3) we obtain the well known expression

$$m(r,t) = 4\pi\int_0^r r^2 T^0_0 dr \quad (21)$$

Now, considering the definition of the Weyl tensor

$$C^{\alpha}_{\beta\gamma\delta} = R^{\alpha}_{\beta\gamma\delta} - \frac{1}{2}R^{\alpha}_{\gamma}g_{\beta\delta} + \frac{1}{2}R_{\beta\gamma}\delta^{\alpha}_{\delta} - \frac{1}{2}R_{\beta\delta}\delta^{\alpha}_{\gamma} + \frac{1}{2}R^{\alpha}_{\delta}g_{\beta\gamma} + \frac{1}{6}R\left(\delta^{\alpha}_{\gamma}g_{\beta\delta} - g_{\beta\gamma}\delta^{\alpha}_{\delta}\right) \tag{22}$$

and eqs. (3), (4), (5), (17) and (19), it follows that

$$m = \frac{4\pi}{3}r^3\left(T^0_0 + T^1_1 - T^2_2\right) + W \tag{23}$$

Differentiating (23) with respect to r and using (21) we get

$$W' = -\frac{4\pi}{3}r^3\left(T^0_0\right)' + \frac{4\pi}{3}\left[r^3\left(T^2_2 - T^1_1\right)\right]' \tag{24}$$

and integrating

$$W = -\frac{4\pi}{3}\int_0^r r^3\left(T^0_0\right)' dr + \frac{4\pi}{3}r^3\left(T^2_2 - T^1_1\right) \tag{25}$$

Finally, inserting (25) into (23) we obtain

$$m(r,t) = \frac{4\pi}{3}r^3 T^0_0 - \frac{4\pi}{3}\int_0^r r^3\left(T^0_0\right)' dr \tag{26}$$

Observe that expressions (23), (25) and (26) are the same for the static (or slowly evolving) case [7]. Of course, in the latter case T^0_0 and $-T^1_1$ denote the proper energy density and radial pressure, which is no longer true in the general case as can be seen from eqs.(7) and (8).
Instead of dealing with the mass function we shall now use the Tolman mass, which, as will be seen in the next section, appears to embody the physical idea of active gravitational mass better than the mass function.

3 The Tolman mass

The Tolman mass for a spherically symmetric distribution of matter is given by (eq.(24) in [8])

$$\begin{aligned} m_T = \quad & 4\pi\int_0^{r_\Sigma} r^2 e^{(\nu+\lambda)/2}\left(T^0_0 - T^1_1 - 2T^2_2\right) dr \\ + \quad & \frac{1}{2}\int_0^{r_\Sigma} r^2 e^{(\nu+\lambda)/2}\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\left[\partial\left(g^{\alpha\beta}\sqrt{-g}\right)/\partial t\right]}\right) g^{\alpha\beta} dr \end{aligned} \tag{27}$$

where L denotes the usual gravitational lagrangian density (eq.(10) in [8]). Although Tolman's formula was introduced as a measure of the total energy of the system, with no commitment to its localization, we shall define the mass within a sphere of radius r, completely inside Σ, as

$$\begin{aligned} m_T = & \quad 4\pi \int_0^r r^2 e^{(\nu+\lambda)/2} \left(T_0^0 - T_1^1 - 2T_2^2 \right) dr \\ + & \quad \frac{1}{2} \int_0^r r^2 e^{(\nu+\lambda)/2} \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \left[\partial \left(g^{\alpha\beta} \sqrt{-g} \right) / \partial t \right]} \right) g^{\alpha\beta} dr \end{aligned} \tag{28}$$

This extension of the global concept of energy to a local level [9] is suggested by the conspicuous role played by m_T as the "effective gravitational mass", which will be exhibited below. Even though Tolman's definition is not without its problems [9, 10], we shall see that m_T, as defined by (28), is a good measure of the active gravitational mass, at least for the system under consideration.

Let us now evaluate expression (28). The first integral in that expression

$$I \equiv 4\pi \int_0^r r^2 e^{(\nu+\lambda)/2} \left(T_0^0 - T_1^1 - 2T_2^2 \right) dr \tag{29}$$

may be transformed as follows. Integrating by parts and using (21), we obtain

$$\begin{aligned} I = & \quad e^{(\nu+\lambda)/2} \left[m(r,t) - \frac{4\pi}{3} r^3 \left(T_1^1 + 2T_2^2 \right) \right] \\ - & \quad \int_0^r e^{(\nu+\lambda)/2} \left(\frac{\nu' + \lambda'}{2} \right) \left[m(r,t) - \frac{4\pi}{3} r^3 \left(T_1^1 + 2T_2^2 \right) \right] dr + \\ + & \quad \int_0^r \frac{4\pi}{3} r^3 e^{(\nu+\lambda)/2} \left[\left(T_1^1 \right)' + 2 \left(T_2^2 \right)' \right] dr \end{aligned} \tag{30}$$

Using the field equations and (15) we then obtain

$$\begin{aligned} I = & \quad e^{(\nu+\lambda)/2} \left[m(r,t) - \frac{4\pi}{3} r^3 T_1^1 \right] \\ - & \quad \int_0^r e^{(\lambda-\nu)/2} \frac{r^2}{2} \left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2} \right) dr \end{aligned} \tag{31}$$

From (eq.(13) in [8])

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\left[\partial\left(g^{\alpha\beta}\sqrt{-g}\right)/\partial t\right]}\right) = -\Gamma^0_{\alpha\beta} + \frac{1}{2}\delta^0_\alpha\Gamma^\sigma_{\beta\sigma} + \frac{1}{2}\delta^0_\beta\Gamma^\sigma_{\alpha\sigma} \tag{32}$$

and so the second integral (II) in (28) may be expressed as

$$II = \frac{1}{2}\int_0^r r^2 e^{(\lambda-\nu)/2}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) dr \tag{33}$$

Thus

$$m_T \equiv I + II = e^{(\nu+\lambda)/2}\left[m(r,t) - 4\pi r^3 T^1_1\right] \tag{34}$$

This is, formally, the same expression for m_T in terms of m and T^1_1 that appears in the static (or quasi-static) case (eq.(25) in [7]).
Replacing T^1_1 by (4) and m by (19) and (20), one also finds

$$m_T = e^{(\nu-\lambda)/2}\,\nu'\,\frac{r^2}{2} \tag{35}$$

This last equation brings out the physical meaning of m_T as the active gravitational mass. Indeed, it can be easily shown [11] that the gravitational acceleration (a) of a test particle, instantaneously at rest in a static gravitational field, as measured with standard rods and coordinate clock is given by

$$a = -\frac{e^{(\nu-\lambda)/2}\,\nu'}{2} = -\frac{m_T}{r^2} \tag{36}$$

A similar conclusion can be obtained by inspection of eq.(16) (valid only in the static or quasi-static case) [12]. In fact, the first term on the right side of this equation (the "gravitational force" term) is a product of the "passive" gravitational mass density $(\rho + P_r)$ and a term proportional to m_T/r^2.
In the next section we shall get another expression for m_T, which appears to be more suitable for the treatment of the problem under consideration. This expression will be evaluated immediately after the system departs from equilibrium, where "immediately" means on a time-scale such that the value of ω remains unchanged. Therefore the physical meaning of m_T, as the active gravitational mass obtained for the static (and quasi-static) case, may be safely extrapolated to the non-static case within this time-scale.

4 Tolman mass, density inhomogeneity and local anisotropy

The required expression for the Tolman mass will be obtained as follows. Taking the r-derivative of (35) we obtain

$$rm'_T = \frac{e^{(\nu-\lambda)/2}}{2} r^3 \left[\frac{\nu'^2}{2} - \frac{\lambda'\nu'}{2} + \nu'' + \frac{2\nu'}{r}\right] \tag{37}$$

On the other hand, it follows from (34) and (23) that

$$3m_T = e^{(\nu+\lambda)} \left[4\pi r^3 \left(T^0_0 - 2T^1_1 - T^2_2\right) + 3W\right] \tag{38}$$

Combining (37) and (38) and using the field equations and (18), we obtain

$$\begin{aligned} rm'_T - 3m_T &= e^{(\nu+\lambda)/2}\left[4\pi r^3\left(T^1_1 - T^2_2\right) - 3W_{(s)}\right] \\ &+ \frac{e^{(\lambda-\nu)/2} r^3}{4}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) \end{aligned} \tag{39}$$

which can be formally integrated to give

$$\begin{aligned} m_T = Cr^3 &+ r^3 \int_0^r \frac{e^{(\nu+\lambda)/2}}{r^4}\left[4\pi r^3\left(T^1_1 - T^2_2\right) - 3W_{(s)}\right] dr \\ &+ r^3 \int_0^r \frac{e^{(\lambda-\nu)/2}}{4r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) dr \end{aligned} \tag{40}$$

or equivalently

$$\begin{aligned} m_T = Cr^3 &+ r^3 \int_0^{r_\Sigma} \frac{e^{(\nu+\lambda)/2}}{r^4}\left[4\pi r^3\left(T^1_1 - T^2_2\right) - 3W_{(s)}\right] dr \\ &+ r^3 \int_0^{r_\Sigma} \frac{e^{(\lambda-\nu)/2}}{4r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) dr \\ &- r^3 \int_r^{r_\Sigma} \frac{e^{(\nu+\lambda)/2}}{r^4}\left[4\pi r^3\left(T^1_1 - T^2_2\right) - 3W_{(s)}\right] dr \\ &- r^3 \int_r^{r_\Sigma} \frac{e^{(\lambda-\nu)/2}}{4r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) dr \end{aligned} \tag{41}$$

Finally, evaluating (40) at r_Σ to obtain C, and replacing it in (41), we obtain, using (18) and (25),

$$\begin{aligned}
m_T &= (m_T)_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 \\
&- r^3 \int_r^{r_\Sigma} e^{(\nu+\lambda)/2} \left[\frac{8\pi}{r}\left(T_1^1 - T_2^2\right) + \frac{1}{r^4}\int_0^r 4\pi\tilde{r}^3 (T_0^0)' d\tilde{r}\right] dr \\
&- r^3 \int_r^{r_\Sigma} \frac{e^{(\lambda-\nu)/2}}{2r}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) dr
\end{aligned} \tag{42}$$

In the static (or quasi-static) case ($\ddot{\lambda} = \dot{\lambda}^2 = \dot{\lambda}\dot{\nu} = 0$) the expression above is identical to eq.(32) in [7].

Let us now assume that our system, initially at rest and in equilibrium, is perturbed and departs from equilibrium. Since ω is the fluid velocity as measured by our local Minkowski observer, then immediately after this departure we have

$$\omega \approx 0 \qquad\qquad \dot{\omega} \neq 0 \tag{43}$$

and, from (6) and (10),

$$\dot{\lambda} \approx 0 \tag{44}$$

On the other hand, the following expression may be easily obtained for $\ddot{\lambda}$ from (6) and (10):

$$\begin{aligned}
\ddot{\lambda} &= -8\pi r e^{(\nu+\lambda)/2}[\,(\rho + P_r)\,\frac{\omega}{1-\omega^2}\frac{\dot{\nu}}{2} + \frac{(\rho+P_r)\,\omega}{1-\omega^2}\frac{\dot{\lambda}}{2} \\
&+ \left(\dot{\rho} + \dot{P}_r\right)\frac{\omega}{1-\omega^2} + (\rho + P_r)\,\dot{\omega}\frac{1+\omega^2}{\left(1-\omega^2\right)^2}\,]
\end{aligned} \tag{45}$$

or, evaluating this immediately after the departure from equilibrium,

$$\ddot{\lambda} = -8\pi r e^{(\nu+\lambda)/2}\left[(\rho + P_r)\,\dot{\omega}\right] \tag{46}$$

Inserting (44) and (46) into (42), we finally obtain, using (7), (8) and (9)

$$
\begin{aligned}
m_T &= (m_T)_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 \\
&+ 4\pi r^3 \int_r^{r_\Sigma} e^{(\nu+\lambda)/2} \left[\frac{2}{r}\left(P_r - P_\perp\right) - \frac{1}{r^4}\int_0^r \tilde{r}^3 \rho' d\tilde{r}\right] dr \\
&+ 4\pi r^3 \int_r^{r_\Sigma} e^{\lambda} \left(\rho + P_r\right) \dot{\omega} dr
\end{aligned} \tag{47}
$$

Note that boundary conditions

$$
m_\Sigma = M \qquad , \qquad \nu_\Sigma = -\lambda_\Sigma \qquad , \qquad [P_r]_\Sigma = 0 \tag{48}
$$

where M is the total mass (the mass parameter in the Scwarzschild metric), imply from (34) and (8)

$$
(m_T)_\Sigma = M + \left(\frac{\rho\omega^2}{1-\omega^2}\right)_\Sigma \tag{49}
$$

Thus, although $(m_T)_\Sigma = M$ in the static (and quasi-static) case and immediately after the departure from equilibrium, this is no longer true in general in the dynamic case.We shall discuss on eq.(47) in the next section.

5 Discussion

Let us asssume that our system consists of pure incoherent dust ($P_r = P_\perp = 0$). Then the value of m_T for a spherical region of radius r within Σ, immediately after the system departs from equilibrium, depends on three different contributions. The first one, $M\left(r/r_\Sigma\right)^3$, is the gravitational mass of a static, homogeneous sphere, of radius r, within Σ. The second contribution, $-4\pi r^3 \int_r^{r_\Sigma} \left(e^{(\nu+\lambda)/2}/r^4\right) \left(\int_0^r \tilde{r}^3 \rho' d\tilde{r}\right) dr$, depends on the inhomogeneity of the energy-density distribution and will be positive if, as required by stability conditions, $\rho' < 0$. The last term $4\pi r^3 \int_r^{r_\Sigma} e^{\lambda}\left(\rho + P_r\right)\dot{\omega} dr$ will contribute negatively in the case of collapse ($\dot{\omega} < 0$). Thus, density inhomogeneity tends to increase the Tolman mass within the sphere of radius r, if $\rho' < 0$, leading thereby (according to (36)) to a faster collapse. This conclusion is also true in the case of slowly collapsing spheres, since (47) is identical to (32) in [7] except for the last term containing $\dot{\omega}$. This last term, in turn,

tends to "stabilize" the system by decreasing (increasing) the Tolman mass in the process of collapse (expansion).

In the case of an anisotropic fluid, the term $4\pi r^3 \int_r^{r_\Sigma} e^{(\nu+\lambda)/2}\,(2/r)\,(P_r - P_\perp)\,dr$ plays a similar role to the density inhomogeneity, with $P_r > P_\perp$ ($P_\perp > P_r$) corresponding to $\rho' < 0$ ($\rho' > 0$). The fact that $P_r > P_\perp$ ($P_\perp > P_r$) leads to a stronger (weaker) collapse is, intuitively, in agreement with eq.(16). This conclusion concerning the influence of local anisotropy is also valid in the slowly evolving regime [7]. However, in this case, we only have information about the system at the moment it departs from equilibrium. A complete description of the evolution of the system, requires a full integration of the field equations.

We have seen so far why and how the pace of collapse is affected by the density inhomogeneity and local anisotropy. Since the former is related to the formation of naked singularities, it could be speculated that local anisotropy might also lead to the formation of naked singularities. Of course, only a specific example could confirm (or discard) this suspicion.

References

[1] Eardley D. M. and Smarr L., 1979, Phys. Rev. D **19**, 2239; Christodoulou D., 1984, Commun. Math. Phys. **93**, 171; Newman R. P. A. C., 1986, Class. Quantum Grav **3**, 527; Waugh B. and Lake K., 1988, Phys. Rev. D **38**,1315; Dwivedi I. H. and Joshi P. S., 1992, Class. Quantum Grav **9**, L69; Joshi P. S. and Dwivedi I. H., 1993, Phys. Rev. D **47**, 5357; Singh T. P. and Joshi P. S., 1996, Class. Quantum Grav **13**, 559.

[2] Oppenheimer J and Snyder H., 1939, Phys. Rev. **56**, 455.

[3] Bondi H., 1964, Proc. R. Soc. London, **A281**, 39.

[4] Bowers R. and Liang E., 1974, Astrophys. J. **188**, 657.

[5] Misner C. and Sharp D., 1964, Phys. Rev. **136**, B571.

[6] Cahill M. and McVittie G., 1970, J. Math. Phys. **11**, 1382.

[7] Herrera L.and Santos N. O., 1995, Gen. Rel. Grav., **27**, 1071.

[8] Tolman R., 1930, Phys. Rev., **35**, 875.

[9] Cooperstock F. I., Sarracino R. S. and Bayin S. S., 1981, J. Phys. A **14**, 181.

[10] Devitt J. and Florides P. S., 1989, Gen. Rel. Grav. **21**, 585.

[11] Grøn Ø., 1985, Phys. Rev. D, **31**, 2129.

[12] Lightman A., Press W., Price R. and Teukolsky S., 1975, Problem Book in Relativity and Gravitation (Princeton University Press, Princeton).